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MEASUREMENT OF PLASMA FLOW RATE WITH A HEAD PRESSURE PIPE

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ABSTRACT: A method of measuring plasma flow rate with a head pressure pipe is examined, as well as a method of measuring static pressure. An MK-5 capacitor microphone is used to record the pressure variations. The measurements can be made in stationary and nonstationary modes. There is an analysis of the effect of various factors on the accuracy of measurement of the plasma flow rate (viscosity at low Reynolds numbers, pressure gradient effect on the displacement of the effective center of the pipe, turbulence, rate of movement of the sensor through the plasma flow). Radial distributions of the pressure head are introduced, conducted in an arc with a channel diameter of 20 mm at a current of 160 A. Possible systematic errors are discussed.

Use of a head pressure pipe is the most widespread of the existing methods for measuring the flow rate of a gas [1, 2]. It has recently been used for measuring plasma flow rate in open streams and stabilized arcs. Water-cooled head pressure pipes located in a certain region of the plasma flow [3-6] and uncooled pipes, drawn rapidly through the plasma [7-9] have been used. The nonstationary method of pressure recording makes it possible to achieve a considerable reduction in the geometrical dimensions of the head pressure pipe and to reduce the area of interference with the plasma flow to a minimum. In addition, the rapid movement of the sensor in the plasma flow makes it possible to perform measurements in streams with high thermal fluxes; this is impossible when using water-cooled pipes. /633*

The present paper discusses a method of measuring the pressure head, the static pressure and the plasma flow rate, analyzing the possible systematic errors related to stationary and nonstationary methods of pressure recording. The measurements were performed in a high-frequency induction discharge, in a stabilized arc and in

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the stream of a plasmatron. The power introduced into the plasma varied from 2 to 10 kV. The argon flow was 3 to 44 l/min. The pressure was recorded by a stationary method (micromanometer with an inclined tube and water-cooled head pressure pipe and by a nonstationary method (capacitor microphone with recording device and uncooled head pressure pipe).

Method of Measurement

The pressure at the forward critical point on a body introduced into the flow is determined by the following equation:

$$P = P_0 + \rho v^2/2, \quad (1)$$

where P and P_0 are the head and static pressures, respectively; ρ is the plasma density and v is the plasma flow rate.

If the plasma density is known, the plasma flow rate can be calculated from measurements of the pressure head. The fittings used for measurements by the nonstationary method are shown in Figure 1. Two types of fittings are used for measuring head pressure: (1) a fitting in the form of a Pitot tube, and (2) a cylindrical fitting with an opening in its side. These two fittings have proven identical as far as measurement accuracy is concerned, but the second design (shown in Figure 1b) made it possible to conduct pressure measurements in a narrow region of the arc, stabilized by the water-cooled walls. Measurements of static pressure were conducted with a disk-shaped fitting (Fig. 1c), which is mounted in the flow in such a way that the flat surface of the disk was parallel to the direction of flow. /634

The MK-5 capacitor microphone was used to record the pressure in the nonstationary case. The microphone was wired into a bridge circuit in the form of a push-pull generator using a dual 6NIP triode [10]. The oscillator circuit of the generator, consisting of inductances L_1 and L_2 connected in series and the microphone capacitances C_m, C_3 and C_4, was connected between the triode control grids. The internal resistances of the triodes and the resistance R_1 in the anode circuit form a bridge, with the oscillograph connected on the diagonal. In the absence of a signal, the voltage of both circuits is tuned to a single frequency (tuning is accomplished by capacitor C_3). The pressure signal produced by the fitting is transmitted to the microphone diaphragm and causes a change in C_m, unbalancing the bridge and causing a signal to appear on the oscillograph. In order to increase the sensitivity of the system, it is necessary to have capacitance C_m greater than the capacitance of the wiring, the capacitance of the connecting cable to the capacitance C_3. It is therefore necessary to mount the microphone housing on the generator panel. Movement of the microphone together with the measuring tube relative to the flow was accomplished with the aid of a swivel bearing, since the latter damped out vibrations.

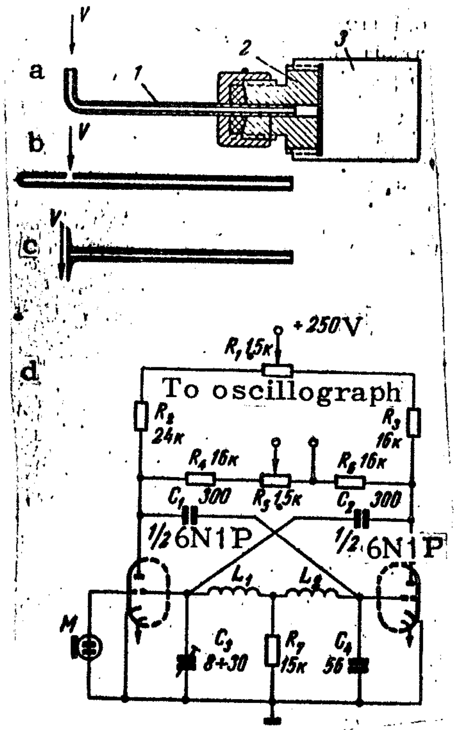

Fig. 1. Measuring Circuit and Fittings: (a, b) Head Pressure; (c) Static Pressure [(1) Fitting; (2) Device Joining Fitting to Microphone; (3) MK-5 Capacitor Microphone]; (d) Measuring Circuit.

To measure the pressure head, it was preferable in some cases to use an acoustic probe (ZA-4), especially where it was not possible to use the microphone with the measurement circuits since in this case the head pressure pipe may be combined with an acoustic probe made of rubber tubing 2.5 m long, thus retaining uniformity of the time curve up to 4000 Hz [11].

Measurement Errors

Let us consider the measurement errors for pressure at a flow rate considerably below the speed of sound, with the Mach number much less than 1 and at atmospheric pressure, when the slippage of the gas is insignificant.

1. Effect of Sensor Inertia (Amplitude and Phase Distortion of the Signal)

As the measuring tube moves through the plasma flow, an alternating pressure develops at its inlet and is transmitted to the diaphragm of the microphone. The output signal will depend to a significant degree on the rate of change in pressure and damping, which is determined mainly by the dimensions of the measuring tube. The time the sensor remains in the plasma must be sufficiently short to ensure preservation of the measuring tube and at the same time must be sufficiently long to ensure uninterrupted recording of the pressure. Amplitude and phase variations of the pressure signal have been discussed in detail in [8, 12, 13]. An optimum ratio was established between the internal diameter of the measuring tube and the space h between the fitting (2, see Fig. 1a) and the diaphragm of the microphone. The optimal value of h, obtained theoretically from the conditions of isentropic compression of the gas, was 0.025 mm [8]; the same article gives the time relationships of the measured pressure for various diameters of the measuring tube and space h. A detailed study of the effect of the resonant frequency of the sensor on measurement accuracy was presented in [13]. It was found that filling the measuring tube with helium shifts the natural frequency of the system toward the region of higher frequencies by a factor of more than two, thus considerably broadening the possible limits of measurement with alternating pressure. /63

To determine the time curve of the measurement system for our case, we recorded the frequency-dependence of the ratio of the measured pressure pulse to the actual value. The time curve was recorded in two ways: by inserting the measuring tube into a jet of air which was interrupted by a rotating disk with a slit, and by placing the tube in the sound field of an oscillating diffusor. In the former case, the actual pressure was determined by connecting the measuring tube to a micromanometer, and in the second case it was determined by measuring the amplitude of the sound pressure, using the microphone without the tube. The experimental points of these two measurements show good agreement. At a frequency of 50 Hz, there was a drop in the measured pressure signal, which became more pronounced with increasing frequency. In the case of measurements in a plasma, the frequency of the changes in the pressure signal were no more than 30 Hz, which ensured uninterrupted reproduction of pressure at the input of the measuring tube.

2. *Effect of Gas Viscosity at Low Reynolds Numbers*

Equation (1), linking the gas pressure and velocity in the vicinity of the measuring opening of the head pressure pipe, was obtained from the solution of the Navier-Stokes equation in the boundary layer without consideration of viscosity. At certain temperatures (for an argon plasma at $T \sim 10^4$ °K) the plasma viscosity exceeds the viscosity of a cold, nonionized gas by more than one order, and at a sufficiently slow flow rate the forces generated by the viscosity become equal to (and in some cases greater than) the inertial forces (Re < 1), so that they must be taken into consideration. This was first determined experimentally by Barker [14]. Homann [15] integrated the Navier-Stokes equation from $d/2 = r$ to infinity for a nongradient flow $v_y = 0$ with $y = 0$, and obtained the following expression for the pressure head:

$$P - P_0 = \frac{\rho v^2}{2} + \mu \int_{\infty}^{r} \frac{\partial^2 v_z}{\partial y^2}\, dz, \qquad (2)$$

where z is a coordinate which coincides with the axis of the measuring tube. Examining the pressure distribution in the boundary layer, Homann [15] estimates the value of the integrand expression in (2):

$$\partial^2 v_z / \partial y^2 = -6vr^2 / z^4$$

and by substitution in (2) obtains

$$(P - P_0) / {}^1/_2 \rho v^2 = 1 + (4 / \mathrm{Re}). \quad (3)$$

Estimating the effect of the thickness of the boundary layer at Re < 100, already comparable to the radius of a cylinder, the author [15] defines (3) more exactly:

$$(P - P_0) / {}^1/_2 \rho v^2 = 1 + [4 / (\mathrm{Re} + 0.457\sqrt{\mathrm{Re}})] = C. \quad (4)$$

This relationship is shown in Figure 2 (Curve 10). The experimental points obtained by Homann for cylinders with external diameters of 1.0, 1.3, 1.19 and 2.0 cm and internal diameters of 0.1 and 0.2 cm show a good fit to the theoretical curve. The data of other authors [8, 14, 16-20] show considerable deviation from theory. This is evidently related to the fact that the derivation of (4) failed to consider the effect of the dimensions of the measuring opening. Figure 2 shows the experimental curves obtained for various ratios of the internal and external diameters of head pressure pipes $\alpha = d_i/d$. As α increases, the curves shift toward the region of large Reynolds numbers. However, the given relationship cannot be established, so that it is necessary to conduct additional careful experiments which will allow the curves shown to be defined more precisely. /63

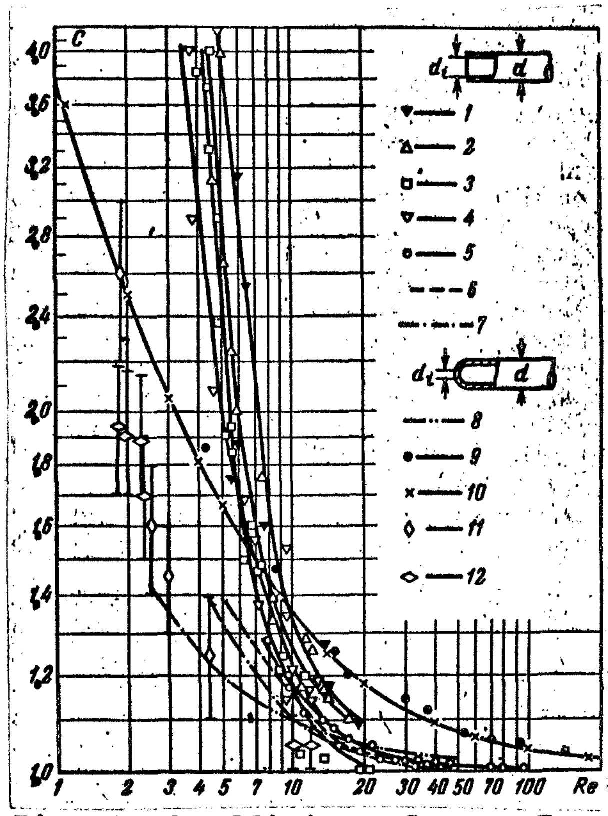

Fig. 2. Coefficient C as a Function of the Reynolds Number for Various Fittings: with a Flat End: (1-4) [19], with α = 0.255, 0.337, 0.530 and 0.685, Respectively; (5) [8], α = 0.64; (6) [14]; (7) [16], α = 0.2; (9, 10) [15]; (11) Measurement in a Stream of Induction Plasma, α = 0.3; (12) Measured in an Arc, α = 0.8.

To determine the corrected curve for the measured pressure, corresponding to the conditions in a plasma, we conducted measurements of the dynamic pressure in the flame of a high-frequency induction torch and in an arc stabilized by water-cooled walls. Measurement of the coefficient C in an induction plasma was achieved by comparing the measured pressure head of the head pressure pipe and the pressure head calculated from the given flow rate of the argon

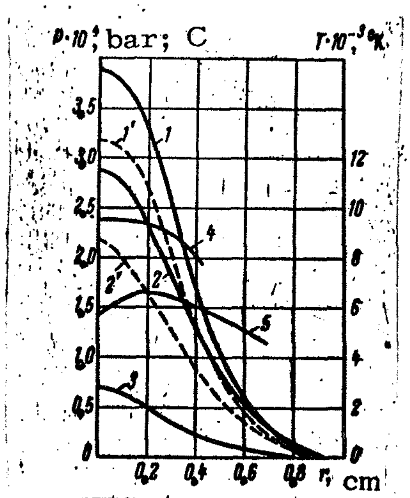

Fig. 3. Radial Distribution of Pressure, Measured by Tubes of Various Diameters in an Arc with a Current of 70 A and an Argon Flow of 15 l/min. (1, 2) Distribution of Head Pressure, Measured by Tubes with Diameters of 0.09 and 0.185, Respectively; (1', 2') Pressure Head Calculated from Curves 1 and 2, with Consideration of Static Pressure (3) and Tapering of the Flow; (4) Plasma Temperature; (5) Excess Pressure Head as Obtained from Measurements with a Small Tube over Those Obtained with a Large Tube.

and the pressure distribution in the flame cross section. Measurement of the coefficient *C* in the arc was accomplished by comparing the pressures measured by tubes of different diameters. Figure 3 shows the pressure distribution along the radius of the arc, as measured by head pressure pipes with diameters of 0.09 and 0.185 cm. The Reynolds numbers were calculated from the radius of the fitting, while the density /63
and viscosity were calculated for the temperature of the inflowing plasma. In view of the fact that the arc is not in equilibrium in its peripheral part [7], the values of *C* were calculated for an area corresponding to the axial portion of the arc. The values which were obtained (see Fig. 3) show good agreement with the measurements made in the flame of an induction torch and with the data in [8], but differ somewhat from the theoretical curve in the region of low Reynolds numbers. The measured values of the coefficient *C* were used in calculating the plasma flow rate in the arc [7] and in a stream obtained with the aid of an induction torch.

3. Effect of Pressure Gradient

As shown in [21, 22], during measurement in a flow with a transverse pressure gradient there is a shift of the effective center of the head pressure pipe relative to its geometric axis in the direction of higher pressures. In gradient flow, the gas moves relative to the intake opening under the influence of the pressure gradient. Let us estimate the value of this term. We will consider that in the vicinity of the forward critical point there is a pressure $P_\delta + y \cdot \mathrm{grad}\, P$ (P_δ being the pressure corresponding to the point of splitting of the flow, and y being the coordinate with its origin at this point and perpendicular to the axis of the head pressure pipe). The transverse velocity will then be:

$$v_y = (y v_z - \sqrt{2\,\mathrm{grad}\,Pd/\rho})\,/\,2, \qquad (5)$$

There is a minus sign in front of the root in this case if grad P is a positive value; if grad P is negative, the sign must be changed. The longitudinal velocity can be expressed as follows:

$$v_z = -\sqrt{2(P_\delta + \operatorname{grad} Py)\rho^{-1}} \tag{6}$$

(since z is directed toward the flow, there is a minus sign in front of the root) and its derivative according to y is equal to

$$\frac{\partial v_z}{\partial y} = \frac{-\operatorname{grad} P}{\sqrt{2\rho(P_\delta + \operatorname{grad} P_y)}}. \tag{7}$$

Then, at the forward critical point with $y = 0$, and assuming grad P to be equal to zero at infinity, we will have

$$\int\limits_r^\infty v_y \frac{\partial v_z}{\partial y} dz = \int\limits_r^\infty \frac{(\operatorname{grad} P)^{3/2}\sqrt{d}}{2\rho\sqrt{P_\delta}} dz = -\frac{(\operatorname{grad} P)^{3/2}\sqrt{d}\, r}{2\rho\sqrt{P_\delta}}.$$

Substituting this expression into the Navier-Stokes equation, we will have (without consideration of the force of viscosity)

$$P_\delta - P_0 = \frac{\rho v^2}{2} + \frac{(\operatorname{grad} Pd)^{3/2}}{4\sqrt{P_\delta}}.$$

From this equation, considering $P_\delta = P + \delta$ grad P (here δ is the shift of the effective center of the pipe relative to its geometric axis), we will have:

$$\frac{\delta}{d} = \frac{1}{4}\sqrt{\frac{\operatorname{grad} Pd}{P + \delta \operatorname{grad} P}}. \tag{8}$$

This relationship, shown in Figure 4, is in satisfactory agreement with the experimental data from [22], although its derivation failed to consider the effect of the measuring opening in the head pressure pipe on the pressure at the braking point. It was found in [22] from measurements in the boundary layer that δ/d = 0.15-0.18. Taking into account the influence of the measuring opening, the following relationship was obtained experimentally in [21]:

$$\delta/d = 0.131 + 0.082 \qquad (9)$$

where d grad P/P = 0.1-1.2.

At low Reynolds numbers, the shift of the effective center of the head pressure pipe increases, but this has not yet been investigated. /638

The transverse motion of the head pressure pipe has an effect on the shift of its effective center. However, this effect is of variable sign, since the shift increases when the pipe is inserted into the flow and decreases when it is withdrawn. Consequently, the pressure distribution along the cross section of the flow is nonsymmetrical, as can be confirmed quite simply.

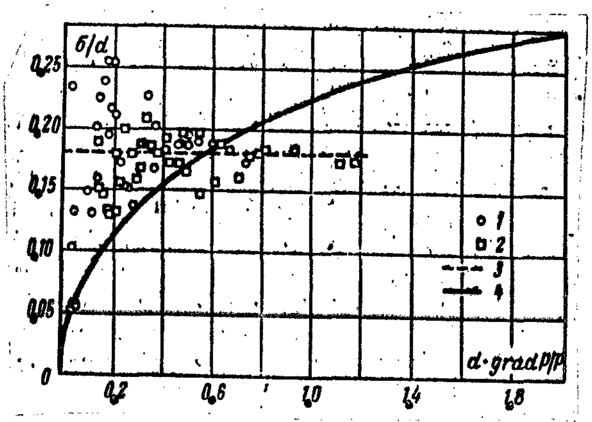

Fig. 4. Shift of the Effective Center of a Head Pressure Pipe as a Function of the Transverse Pressure Gradient: (1-2) Measurements at Velocities of 18.3 and 22.9 m/sec, Respectively; (3) Resultant Straight Line from the Data in [21]; (4) Relationship (8), Obtained from the Solution of the Navier-Stokes Equation.

The effect of the longitudinal pressure gradient makes itself markedly known only at low Reynolds numbers. If we know the thickness of the boundary layer δ, we can estimate the pressure increase, which is determined by the following relationship: $P + \delta$ grad P. According to the data in [15], the thickness of the boundary layer is

$$\delta = 0.457r/\sqrt{\mathrm{Re}} = 0.457d/2\sqrt{\mathrm{Re}}.$$

Therefore

$$\frac{\Delta P}{P} = \frac{0.228}{\sqrt{\mathrm{Re}}}\,\frac{d\,\mathrm{grad}\,P}{P}.$$

In measurements made in an arc, stabilized by water-cooled walls, the maximum longitudinal pressure gradient was found to be about 5000 dyn/cm^2/cm. The measurement error with d = 0.1 cm and Re $\approx$ 10 does not exceed 1%.

4. *Effect of Turbulence in the Flow*

Turbulence in the gas flow can increase considerably the pressure measured by a head pressure pipe. Goldstein [23], on the basis of a theory of isotropic turbulence, obtained the following expressions for the measured pressures:

Full pressure:

$$P = P_0' + {}^1\!/_2\rho v^2 + {}^1\!/_2\rho\overline{(v')^2}$$

Static pressure:

$$P_0' = P_0 + {}^1\!/_6\rho\overline{(v')^2}.$$

Fage [24], on the basis of the measurements made in [25], calculated the static pressure, which was found to be

$$P_0' = P_0 + {}^1\!/_4\rho\overline{(v')^2}.$$

As mentioned above, turbulence increases not only the head pressure but the static pressure as well. Therefore, in measurements in turbulent flows which are conducted in the peripheral regions of an arc and in plasma streams where the degree of turbulence of the flow is close to unity, the measurement error can reach 100%.

Measurement Results

As we mentioned earlier, measurements of the plasma flow rate were made in flows obtained with the aid of induction flames and an arc plasmatron, as well as in an arc stabilized by water-cooled walls. Some results of these measurements of velocity were published in [3-5, 7, 26]. Figures 5 and 6 show the most characteristic pressure distributions, which make it possible to find the systematic errors which arise in the measurement of plasma flow rates with head pressure pipes. Figure 5 shows the radial distributions of /639
head pressure, static pressure and temperature for the cathode region of the arc, stabilized by water-cooled walls and characterized by high pressure gradients. The same figure also shows the systematic errors caused by the shift of the effective center of the head pressure pipe (ΔP_δ) and the effect of the force of viscosity (ΔP_{Re}). The shift of the effective center of the head pressure pipe in the region of maximum transverse pressure gradient causes a 30% increase of the measured pressure relative to the true value. The error (ΔP_{Re}) characterizing the effect of the forces of viscosity at low Reynolds numbers is a function of the high temperature of the cathode region of the arc and amounts to 10-15%. Consequently, in making measurements in plasma flows with high temperature and low gas flow rate, where Re < 10, the measurement errors becomes significant. In the case of measurements in high-speed flows, characteristic of the plasma streams of arc plasmatrons, the errors produced by the presence of viscosity forces become negligibly small, since Re > 20 (Fig. 6). However, the errors produced by the shift of the effective center of the head pressure pipe remain, although they decrease somewhat. The static pressure in the plasma stream, according to measurements with a disk fitting, were equal to zero. Consequently, the pressure measured in the plasma stream in zones close to the axis is the dynamic pressure and makes it possible to calculate the gas flow rate directly [using (1)]. In the peripheral

zones of the plasma stream, where there is mixing of the gas forming the plasma with the surrounding air, turbulence arises, producing additional pressure; therefore, the accuracy of determination of the plasma flow rate decreases.

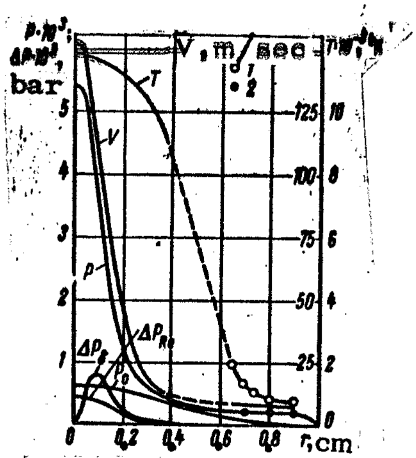

Fig. 5. Radial Temperature Distributions, Head and Static Pressure, Velocity, Systematic Errors ΔP_{δ} and ΔP_{Re}, Produced by Shifting of the Effective Center of the Head Pressure Pipe and the Viscosity of the Plasma, Respectively, Measured at a Distance of 0.8 cm from the Cathode (Arc with a Current of 160 A in a Channel 4.5 cm Long and 2 cm in Diameter, with an Argon Flow of 45 l/min). (1) Temperature Measured with a Chromel-Alumel Thermocouple; (2) Pressure Measured with a Stationary Head Pressure Pipe.

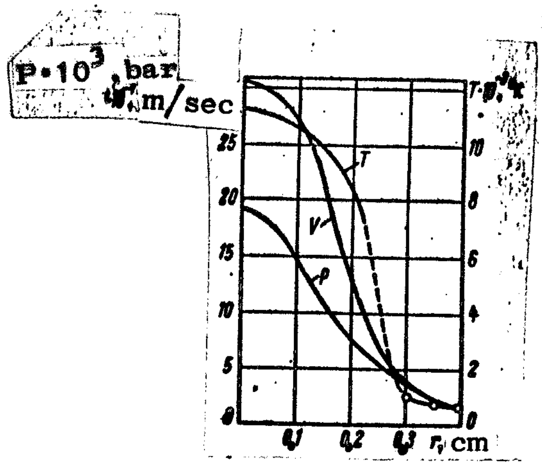

Fig. 6. Radial Distribution of Temperature, Pressure and Flow Rate of Argon Plasma at a Distance of 2 mm from the Nozzle Opening of an Arc Plasmatron (Nozzle Diameter = 7 mm); Points Show Temperature as Determined by a Chromel-Alumel Thermocouple.

From the above, it follows that measurement of the plasma flow rate by a head pressure pipe with an accuracy of at least 5% (without consideration of the above errors) may be conducted only in flows with a high gas flow rate (> 200 m/sec). In measurements under other conditions, all systematic errors must be carefully considered to obtain the desired accuracy. /64

In conclusion, we must emphasize the influence of a strong nonisothermicity of the plasma in the boundary layer at the inlet opening of the fitting, partially accounted for by determinant criteria. Measurements in the stream of an arc plasmatron with the aid of a disk fitting (Fig. 1c) have shown that the effect of nonisothermicity on the measured pressure is at the level of error of the measurement system, and does not exceed 5%. This value is in good agreement with the data in [8, 9, 27], thus making it possible to operate with the relationships for the isothermal case.

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